Mid-Chapter Quiz: Lessons 5-1 through 5-4

Simplify. Assume that no variable equals 0.

1. $(3x^2y^{-3})(-2x^3y^5)$

SOLUTION:

$$(3x^2y^{-3})(-2x^3y^5) = -6x^{2+3}y^{-3+5} \quad \text{Product of Powers}$$
$$= -6x^5y^2 \quad \text{Simplify.}$$

2. $4t(3rt - r)$

SOLUTION:

$$4t(3rt - r) = (4t)(3rt) - (4t)(r) \quad \text{Distributive Property}$$
$$= 12rt^2 - 4rt \quad \text{Simplify.}$$

3. $\dfrac{3a^4b^3c}{6a^2b^5c^3}$

SOLUTION:

$$\frac{3a^4b^3c}{6a^2b^5c^3} = \frac{1}{2}a^{4-2}b^{3-5}c^{1-3} \quad \text{Quotient of Powers}$$
$$= \frac{1}{2}a^2b^{-2}c^{-2} \quad \text{Simplify.}$$
$$= \frac{a^2}{2b^2c^2} \quad \text{Simplify.}$$

4. $\left(\dfrac{p^2r^3}{pr^4}\right)^2$

SOLUTION:

$$\left(\frac{p^2r^3}{pr^4}\right)^2 = \left(p^{2-1}r^{3-4}\right)^2 \quad \text{Quotient of Powers}$$
$$= \left(pr^{-1}\right)^2 \quad \text{Simplify.}$$
$$= p^2r^{-2} \quad \text{Power of a Power}$$
$$= \frac{p^2}{r^2} \quad \text{Simplify.}$$

5. $(4m^2 - 6m + 5) - (6m^2 + 3m - 1)$

SOLUTION:

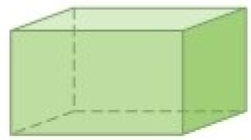
$$(4m^2 - 6m + 5) - (6m^2 + 3m - 1)$$
$$= 4m^2 - 6m + 5 - 6m^2 - 3m + 1 \quad \text{Distributive Property}$$
$$= -2m^2 - 9m + 6 \quad \text{Simplify.}$$

6. $(x + y)(x^2 + 2xy - y^2)$

SOLUTION:

$$(x+y)(x^2+2xy-y^2)$$
$$= x(x^2) + x(2xy) - x(y^2) + y(x^2) + y(2xy) - y(y^2) \quad \text{Distributive Property}$$
$$= x^3 + 2x^2y - xy^2 + yx^2 + 2xy^2 - y^3 \quad \text{Combine like terms.}$$
$$= x^3 + 3x^2y + xy^2 - y^3 \quad \text{Simplify.}$$

7. **MULTIPLE CHOICE** The volume of the rectangular prism is $6x^3 + 19x^2 + 2x - 3$. Which polynomial expression represents the area of the base?



A $6x^4 + 37x^3 + 59x^2 + 3x - 9$
B $6x2 + x + 1$
C $6x^2 + x - 1$
D $6x + 1$

SOLUTION:

$$\text{Base area of a prism} = \frac{\text{Volume}}{\text{Height}}$$

$$A = \frac{6x^3+19x^2+2x-3}{x+3} \quad \frac{\text{Volume}}{\text{Height}}$$
$$= \frac{(x+3)(6x^2+x-1)}{x+3} \quad \text{Factor.}$$
$$= 6x^2 + x - 1 \quad \text{Simplify.}$$

The correct choice is C.

Simplify.

8. $(4r^3 - 8r^2 - 13r + 20) \div (2r - 5)$

SOLUTION:

Use long division.

$$\begin{array}{r} 2r^2 + r - 4 \\ 2r-5\overline{)4r^3 - 8r^2 - 13r + 20} \\ \underline{4r^3 - 10r^2} \\ 2r^2 - 13r \\ \underline{2r^2 - 5r} \\ -8r + 20 \\ \underline{-8r + 20} \\ 0 \end{array}$$

$$\frac{4r^3 - 8r^2 - 13r + 20}{2r - 5} = 2r^2 + r - 4$$

9. $\frac{3x^3 - 16x^2 + 9x - 24}{x - 5}$

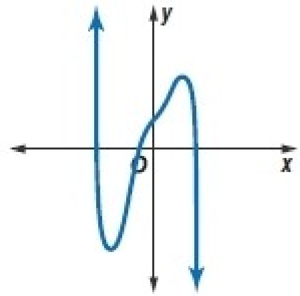
SOLUTION:

Use synthetic division.

$$\begin{array}{r|rrrr} \underline{5} & 3 & -16 & 9 & -24 \\ & & 15 & -5 & 20 \\ \hline & 3 & -1 & 4 & |-4 \end{array}$$

$$\frac{3x^3 - 16x^2 + 9x - 24}{x - 5} = 3x^2 - x + 4 - \frac{4}{x - 5}$$

10. Describe the end behavior of the graph. Then determine whether it represents an odd-degree or an even-degree polynomial function and state the number of real zeros.



SOLUTION:

The end behavior of the graph is:

$f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$

Since the end behavior is in the opposite direction, it is an odd-degree polynomial function.
The graph intersects the x-axis at three points, so there are three real zeros.

11. **MULTIPLE CHOICE** Find $p(-3)$ if $p(x) = \frac{2}{3}x^3 + \frac{1}{3}x^2 - 5x$.

F 0
G 11
H 30
J 36

SOLUTION:

Substitute –3 for x in the equation and then solve.

$$\begin{aligned} p(-3) &= \frac{2}{3}(-3)^3 + \frac{1}{3}(-3)^2 - 5(-3) \\ &= -18 + 3 + 15 \\ &= 0 \end{aligned}$$

The correct choice is F.

12. **PENDULUMS** The formula $L(t) = \frac{8t^2}{\pi^2}$ can be used to find the length of a pendulum in feet when it swings back and forth in *t* seconds. Find the length of a pendulum that makes one complete swing in 4 seconds.

SOLUTION:

Substitute $t = 4$ in the formula.
The length of the pendulum for a complete swing in 4 seconds:

$$L(3) = \frac{8(4)^2}{\pi^2}$$

Use a calculator to simplify.

$$L(3) \approx 12.97 \text{ ft}$$

13. **MULTIPLE CHOICE** Find $3f(a-4) - 2h(a)$ if $f(x) = x^2 + 3x$ and $h(x) = 2x^2 - 3x + 5$.

A $-a^2 + 15a - 74$

B $-a^2 - 2a - 1$

C $a^2 + 9a - 2$

D $-a^2 - 9a + 2$

SOLUTION:

$$\begin{aligned} & 3f(a-4) - 2h(a) \\ &= 3\left[(a-4)^2 + 3(a-4)\right] - 2\left[2a^2 - 3a + 5\right] && \text{Substitute: } f(a-4) = (a-4)^2 + 3(a-4),\ h(a) = 2a^2 - 3a + 5 \\ &= 3\left[a^2 - 8a + 16 + 3a - 12\right] - 4a^2 + 6a - 10 && \text{Simplify.} \\ &= 3a^2 - 15a + 12 - 4a^2 + 6a - 10 && \text{Distributive Property, combine like terms} \\ &= -a^2 - 9a + 2 && \text{Simplify.} \end{aligned}$$

The correct choice is D.

14. **ENERGY** The power generated by a windmill is a function of the speed of the wind. The approximate power is given by the function $P(s) = \frac{s^3}{1000}$, where s represents the speed of the wind in kilometers per hour. Find the units of power $P(s)$ generated by a windmill when the wind speed is 18 kilometers per hour.

SOLUTION:

Substitute $s = 18$ kilometres in the expression.
The unit of power generated by a wind mill is:

$$\begin{aligned} P(18) &= \frac{18^3}{1000} \\ &= 5.832 \text{ units} \end{aligned}$$

Use $f(x) = x^3 - 2x^2 - 3x$ for Exercises 15–17.

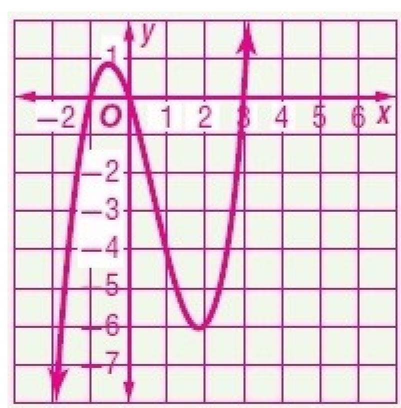
15. Graph the function.

SOLUTION:

Substitute the random values for *x* and determine the corresponding *y*-values. Then, tabulate the values.

x	$f(x)$
−2	−10
−1	0
0	0
1	−4
2	−6
3	0

Graph the function.



16. Estimate the *x*-coordinates at which the relative maxima and relative minima occur.

SOLUTION:

The relative maximum is the point of the function with the greatest *y*-coordinate. The relative minimum is the point with the least *y*-coordinate.

Relative maxima: $x = –0.5$.
Relative minima: $x = 2$.

17. State the domain and range of the function.

SOLUTION:

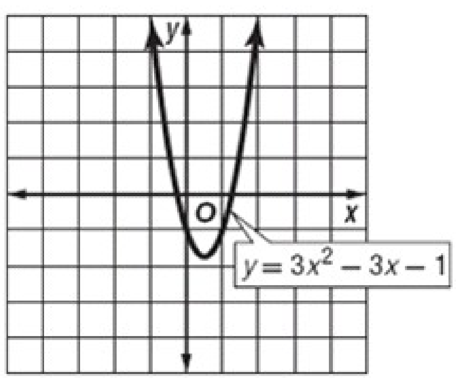
The domain is the set of all *x*-coordinates and the range is the set of all *y*-coordinates of the ordered pairs of the function.

D = {all real numbers};
R = {all real numbers};

18. Determine the consecutive integer values of *x* between which each real zero is located for $f(x) = 3x^2 – 3x – 1$.

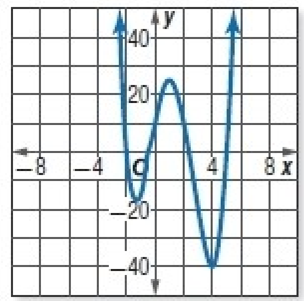
SOLUTION:

Graph the function.



The graph intersects the *x*-axis between −1 and 0 and between 1 and 2.

Refer to the graph.



19. Estimate the *x*-coordinate of every turning point, and determine if those coordinates are relative maxima or relative minima.

SOLUTION:

The *x*-coordinates of the turning points are: –1.5, 1 and 4.
Relative maxima at $x \approx 1$;
Relative minima at $x \approx –1.5$ and $x \approx 4$;

20. Estimate the *x*-coordinate of every zero.

SOLUTION:

The graph intersects the *x*-axis at about –2, –0.5, 2.5 and 5.

21. What is the least possible degree of the function?

SOLUTION:

Since the graph intersects the *x*-axis at 4 points, the degree of the polynomial function is 4.